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Efficiency limits of diffusive shock acceleration

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Abstract: It is well accepted today that diffusive acceleration in shocks results to the cosmic ray spectrum formation. This is in principle true for non-relativistic shocks, since there is a detailed theory covering a large range of their properties and the resulting power-law spectrum, which is nevertheless not as efficient to reach the very high energies observed in the cosmic ray spectrum. On the other hand, the cosmic ray maximum energy and the resulting spectra from relativistic shocks, are still under investigation and debate concerning their contribution to the features of the cosmic ray spectrum and the measured, or implied, cosmic ray radiation from candidate astrophysical sources. Here, we discuss the efficiency of the first order Fermi (diffusive) acceleration mechanism up to relativistic shock speeds, presenting Monte Carlo simulations.

Introduction

The notion that collisionless plasma shocks can accelerate charged particles into high energies resulting into the observed cosmic ray spectrum, has been around for many years and by now it is widely accepted that this process can account for the majority of the energetic, non-thermal particle distributions that we infer in various astrophysical environments. The work of the late 70s by a number of authors (e.g. [5], [1], [2]) established in principle the basic mechanism of particle diffusive acceleration in non-relativistic shocks. Since then, considerable work has been done analytically and numerically on the subject, however questions still need to be answered on this acceleration mechanism especially at relativistic shock speeds.

In the present paper we will present simulations from non-relativistic up to relativistic shock speeds, in order to test the acceleration efficiency of the first order Fermi mechanism. More precisely, these simulations include sub-luminal and super-luminal shock acceleration with an aim to draw conclusions on the primary spectra of the relevant astrophysical sources such as Supernovae Remnants, Active Galactic Nuclei hot spots and Gamma Ray Bursts.

Numerical simulations

The purpose of using a Monte Carlo code is to simulate the particle transport equation, in our case for mild and relativistic shock velocities. The appropriate time independent Boltzmann equation is given by

$$\Gamma(V + \upsilon\mu)\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t}|_c$$

where V is the fluid velocity, υ the velocity of the particle, Γ the Lorentz factor of the fluid frame, μ the cosine of the particle's pitch angle and $\frac{\partial f}{\partial t}|_c$ the collision operator. The frames of reference used in the code are the local fluid frame, the normal shock frame and the de Hoffman-Teller frame. Pitch angle is measured in the local fluid frame, while the value x is the distance of the particles to the shock front, where the shock is assumed to be placed at $x = 0$. Specifically, in order for the above equation to be solved by the Monte Carlo technique, we assume that i) the collisions represent scattering in pitch angle and ii) the scattering is elastic in the fluid frame. We begin the simulation by injecting 10^5 particles far upstream. First order Fermi (diffusive) acceleration can be simulated provided that the particles' guidance center undergoes multiple scatterings with the assumed magnetized media and in each shock crossing the parti-
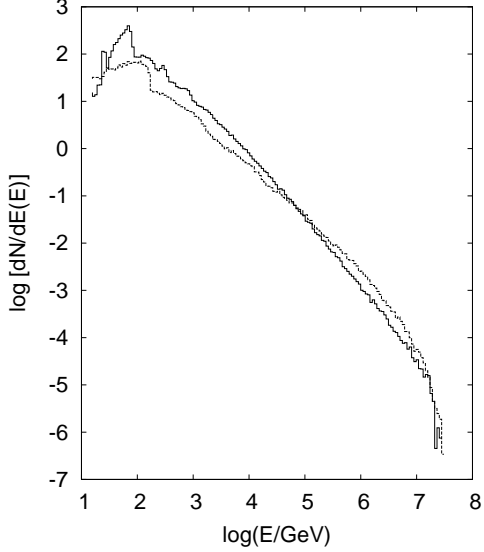
Figure 1: Two sub-luminal shock spectra, one for a non-relativistic shock of a gamma factor equal to 1.0001 (i.e. V=0.01c) and the other for a gamma equal to 5.0 (i.e. V=0.98c) with an inclination angle of 89 and 39 degrees respectively. One can see the almost perfect superposition of the spectra, showing the equivalence of acceleration efficiency between the non-relativistic *perpendicular* shock and the relativistic sub-luminal oblique one.

Figure 2: Super-luminal shock spectrum for a gamma factor of 10 at 89 degrees.

cles gain an amount of energy. The mean free path λ is calculated in the respective fluid frames (i.e. upstream or downstream) assuming a momentum dependence to the mean free path of the particle such as, $\lambda = \lambda_o \cdot p$, where $\lambda_o = 10 \cdot r_g$, which is related to the spatial diffusion coefficient, κ. Specifically for oblique shocks, in the shock normal which lies in the x direction, the diffusion coefficient κ, equals to $\kappa_\parallel cos^2\psi$, where $\kappa_\parallel = \lambda v/3$ and ψ is the inclination of the shock to the magnetic field lines. A pitch angle diffusion algorithm is used (e.g. see [6]). All results are given at the downstream side of the shock frame. In general, flow into and out of the shock discontinuity is not along the shock normal, but a transformation is possible into the shock frame to render the flows along the normal (e.g. [3]) and for simplicity we assume that such a transformation has already been made. During o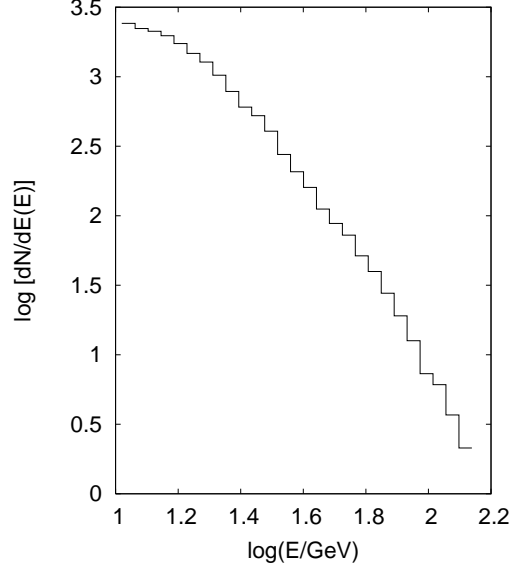ur simulations continuous Lorentz transformations are performed from the local fluid rest frames to the shock frame to check for shock crossings.

Furthermore, for the super-luminal shock conditions (see [6]) we consider a helical trajectory motion of each test-particle of momentum p, in the fluid frames upstream or downstream respectively, where the velocity coordinates (v_x, v_y, v_z) of the particle are calculated in a three dimensional space. In principle, we follow the helix trajectory of each particle in time t, where t is the time from detecting the shock intersection at x, y, z. Further details of this method can be found in the work mentioned above. All particles leave the system if they escape far downstream at the spatial boundary r_b, or alternatively, if they reach a specified maximum energy E_{max}, for computational convenience, even though other physical parameters describing particle escape or energy loss would need to be taken into account in more realistic situations. The downstream spatial boundary required, can be initially estimated from the solution of the convection-diffusion equation in a non-relativistic, large-angle scattering approximation in the downstream plasma which gives the chance of return to the shock as, $exp(-V_2 r_b/x_i)$. In fact, we have per-
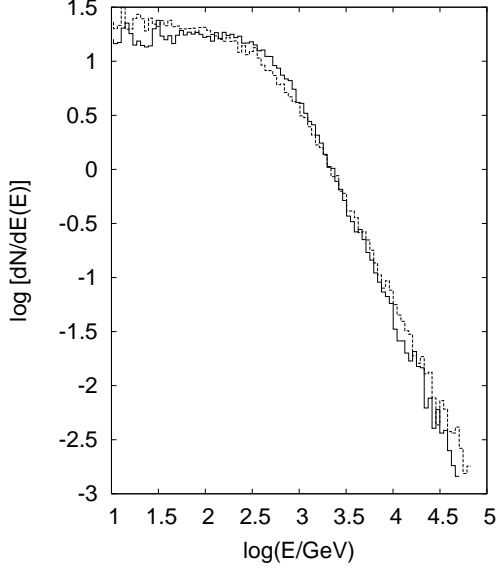
Figure 3: Two super-luminal shock spectra for a gamma factor of 500, one for an inclination angle of 50 degrees and the other for an inclination of 89 degrees. The spectra reach a maximum of 10^{14}eV for protons. This cut-off in the energy is due to the stream flow, sweeping the particles downstream limiting their chance of returning back to the shock.
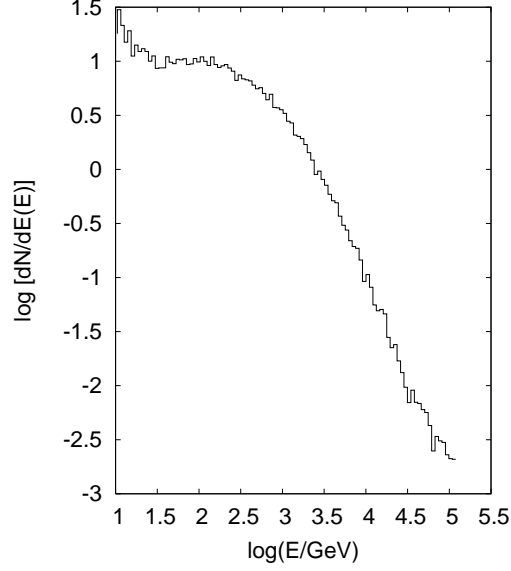

Figure 4: A super-luminal shock spectrum for a gamma factor of 900 at 75 degrees. One sees (simulated spectra for other shock inclinations are comparable) the smooth power-law formation of the spectrum. Nevertheless, even for such high Lorentz factor the super-luminal shocks do not seem to be as efficient as the sub-luminal ones (e.g. [6]), since the simulations showed that almost all of the particles are advected downstream just after a shock cycle.

formed many runs with different spatial boundaries to investigate the effect of the size of the acceleration region on the spectrum, as well as to find a region where the spectrum is size independent. We note that in the pitch angle diffusion case, the inherent anisotropy due to the high downstream sweeping effect may greatly modify this analytical estimate.

Results

In this section we present results of the simulations shown in the figures 1-4.

Two sub-luminal shock spectra are shown in figure 1, one for a non-relativistic shock of a gamma factor equal to 1.0001 (i.e. V=0.01c) and the other for a gamma equal to 5.0 (i.e. V=0.98c) with an inclination angle of 89 and 39 degrees respectively. One can see the almost perfect super-

position of the spectra, showing the equivalence of the acceleration efficiency between the non-relativistic *perpendicular* shock and the relativistic sub-luminal oblique one. The above correlation of efficiency between non-relativistic perpendicular shocks and mild relativistic oblique ones, applies to many other shock spectra we simulated, as long as the non-relativistic shocks are nearly perpendicular. For the case shown here, the maximum energy corresponds to $\sim 10^{7.5}$GeV for protons. Adding to the above, [9] have shown that non-relativistic perpendicular shocks, are efficient accelerators for cosmic rays reaching energies as high as $\sim 10^{17}$eV, since the acceleration in those shocks seems to be faster under certain diffusion conditions [4]. Also as [6] have shown, sub-luminal highly relativistic shocks are the most efficient accelerators comparing to the above, result-

ing in an average cosmic ray energy gain of $\sim \Gamma^{3.4}$ after two complete shock cycles. For this case, Γ is the boost factor of the shock, while in the simulations the test-particles are considered already relativistic with an initial Lorentz gamma, $\gamma = \Gamma+100$. One further result of the simulations, is that for oblique shocks below a Lorentz factor of $\Gamma=5$ (i.e. $V=0.98c$) as higher the inclination of the shock as greater its efficiency to accelerate particles up to 10^{17}eV (i.e. protons, iron nuclei).

In figure 2 we show a super-luminal shock spectrum for a gamma factor of 10 at 89 degrees. In figure 3 one sees two super-luminal shock spectra for a gamma factor of 500, one for an inclination angle of 50 degrees and the other for an inclination of 89 degrees. Simulations show that the inclination of the shock in the super-luminal cases does not change the results. The cut-off is due to the flow, sweeping the particles downstream limiting their chance to return back to the shock, diminishing the chances for these shocks to be efficient cosmic ray accelerators. In figure 4, a super-luminal shock spectrum for shock gamma factor of 900 at 75 degrees is shown. One sees the smooth power-law formation of the above spectra (in contrast to the sub-luminal spectra formations in [6]). Nevertheless, even for as high Lorentz gamma as 900, the accelerated particles do not reach very high energies (e.g. 10^{14}eV for protons) since almost all of them are advected downstream, after their helix trajectory performs a shock-cycle crossing. All the simulated super-luminal shock spectra can be well fitted by a power-law with a spectral index ranging between $\sim 2.0 - 2.3$ in contrast to the work of [6] concerning sub-luminal relativistic shocks. The latter spectra appear flatter, at the highest speeds with a characteristic plateau-like structure, since first, a highly relativistic shock catches the particles up within less a shock cycle, which does not allow sufficient time for isotropisation and second, because relativistic shocks are sensitive to the applied particle scatter model [8] .

Conclusions

We discussed the mechanism of diffusive shock acceleration, presenting simulations in order to test the efficiency of sub-luminal and super-luminal relativistic shocks in a quantitative comparison to non-relativistic ones. It is certain that super-luminal shocks are not as efficient first order Fermi cosmic ray accelerators as the sub-luminal ones. Nevertheless, the super-luminal shocks can be well fitted by a power-law with a spectral index ranging between $\sim 2.0 - 2.3$ in contrast to the relativistic sub-luminal ones which give flatter spectra. Specifically, the super-luminal shocks are efficient in accelerating particles up to a maximum energy of around 10^{14}eV (i.e. protons or iron nuclei). On the other hand, the sub-luminal relativistic oblique shocks are better accelerators concerning a maximum energy above 10^{18}eV, as shown in similar works as well, but the spectral features are not as smooth as in super-luminal ones (which are given by a 'clean' power-law), indicating a strong connection to the kinematical details of the particle diffusion while crossing the relativistic shock front. Interestingly, it is shown that perpendicular non-relativistic shocks seem to be as efficient accelerators as sub-luminal mild relativistic ones. There is work under way concerning the initial parameters which could affect the shock behaviour in highly relativistic speeds.

Acknowledgements

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